International Journal of Economics and Financial Issues

ISSN: 2146-4138

available at http: www.econjournals.com

International Journal of Economics and Financial Issues, 2015, 5(2), 515-522.

Chaotic Structure of the BRIC Countries and Turkey's Stock Market

Samet Günay*

Department of Banking and Finance, School of Applied Sciences, Istanbul Arel University, Istanbul, Turkey.
*Email: dr.sgunay@gmail.com

ABSTRACT

In this study, the parameters of chaos are analyzed for the leading emerging stock markets: Brazil, Russia, India, China, and Turkey (BRIC-T). As chaos has properties such as nonlinearity, sensitivity to initial conditions, and fractality, we performed different methods to identify the existence of the chaos in stock index returns of the BRIC-T countries, using the Brock-Dechert-Scheinkman test, the Largest Lyapunov exponent and the Box-Counting method. Although there is widespread interest in chaos in finance theory, previous studies have neglected the long memory issue in their filtering model of nonlinear behaviors. Due to the fact that the Rescaled Range (R/S) analysis and Smith's (2005) modified GPH test indicated long memory in the index returns, we filtered the linear structure of the returns using the methods (ARFIMA, FIGARCH, FIEGARCH) which take long memory into account. Though the results have some significant evidence of chaos, the findings are too weak to support the presence of chaos in the stock markets of BRIC-T countries.

Keywords: Chaos, Fractals, Largest Lyapunov Exponent, Brock-Dechert-Scheinkman Test, Fractal Dimension
JEL Classifications: C14, C22, G10

1. INTRODUCTION

As stated by Williams (1997), chaos is a continuous and irregular long-term evaluation that satisfies specific mathematical criteria and arises from deterministic nonlinear systems. In a chaotic system, the trajectory of phase space may have fractal features. Also, in time series literature, chaos satisfies the criteria that autocorrelation function approaches zero in finite time, as trajectories are not constant (Brown, 1997).

Deterministic processes can be defined as processes that yield exactly the same outputs in every repetition in identical conditions. In contrast, stochastic processes yield different outputs in every repetition. Deterministic systems demonstrate regular behaviors, and cases where the deterministic model is known, the future behavior of the system can be predicted. While most systems display regular and foreseeable behaviors, some deterministic systems exhibit irregular and random-looking behaviors, and these are defined as chaotic systems (Ban and Shachrnurove, 2002). Although nonlinear relationships are inconvenient for most of us, it is clear that nature does not generally exhibit linear relationships. In practice researchers prefer to transform nonlinear relationships into linear forms by converting data (e.g. taking logarithm). This application only provides graphical or analytical simplicity, yet does not change the nonlinear reality (Williams, 1997). Owing to the high sensitivity to both initial conditions and parameter changes, long-term estimations are fruitless in chaotic systems. However, sensitivity to the initial conditions and parameters does not preclude the possibility of producing predictions with reasonable accuracy. This can be attributed to the fact that, though the initial conditions are quite different, the time series produced from chaotic systems are duplicative in the preliminary stages (Cuthbertson, 1999).

Linear models produce only four types of behaviors; oscillatory and stable, oscillatory and explosive, non-oscillatory and stable, non-oscillatory and explosive. Nonlinear models, however, produce many variations. For example, the system may have instantaneous volatility explosions and large movements that are rarely seen. Stock market analysts and investors have always been interested in these types of events, as Black Monday of 1987 being a prime example (Hsieh, 1991). Chaotic behaviors are now a given in financial markets, so new models and theories that take chaos and its features into account must be produced. Studies that analyze chaos are essential, as they reveal the true character of financial markets. Although there is great interest in chaos in literature, these studies generally focus on some specific features of chaos at the expense of others.

Seeking to improve on finance literature, in this study we examine the chaotic structure of stock markets from emerging countries such as Brazil, Russia, India, China and Turkey (BRIC-T). Another innovation of this study is that long memory features of the time series used in the filtering models are thaken into account prior to Brock-Dechert-Scheinkman (BDS) test. The filtration process was conducted with ARFIMA, FIGARCH and FIEGARCH models. While BDS test and Largest Lyapunov exponent are popular methods in chaos analysis, fractality, which is another parameter of chaos, is rarely considered in the empirical modelling in literature. Therefore, in addition to the BDS test and Largest Lyapunov exponent we conducted fractality tests using Box-Counting method to investigate the determinants of chaos from a wider perspective.

2. LITERATURE REVIEW

The studies of Takens (1981) and Grassberger and Procaccia (1983) were the preliminary studies about chaos and the existence of nonlinear dynamics. To reveal the presence of chaos, Grassberger and Procaccia (1983) used fractal approximation. They stated that the correlation exponent v has a close relationship with the fractal dimension, and that nonlinear dependencies can generate chaos. The initial studies regarding financial and economic time series were conducted by Hinich and Patterson (1985), Brockett et al. (1988), Brock (1986), Brock et al. (1987), Barnett and Chen (1988), Scheinkman and Lebaron (1989) and Hsieh (1989). Brock (1986) demonstrated the presence of chaos in the U.S. Gross National Product data using the correlation dimension and the largest Lyapunov exponent, while Brock et al. (1987) introduced the BDS test, based on the correlation dimension, and applied it to residuals of financial time series models. The test was modified in 1996 by Brock et al. (1996). While there are different methods in literature for revealing nonlinearities in the financial time series, the BDS test has gained wide acceptance. Scheinkman and Lebaron (1989) developed an algorithm for distinguishing stochastic and deterministic systems, and also demonstrating the presence of nonlinear dependence in the weekly returns of the Center for Research in Security Prices data. Similarly, Lui et al. (2001) used the Grassberger-Procaccia correlation exponent and the BDS test to discriminate stochastic and deterministic systems. They also stated that the BDS test properly rejected the i.i.d null hypothesis when the data exhibited deterministic chaos property. Barnett and Chen (1988) displayed the evidence of chaos in certain monetary aggregates and DeCoster and Mitchell (1991) obtained findings in Divisia M2, Divisia M3, and simple sum M2 data that supported the nonlinearity results of Barnett and Chen (1988). Hsieh (1989) analyzed the nonlinearity of five major foreign exchange rates, and demonstrated the existence of substantial nonlinearity in a multiplicative rather than additive form. He also stated that the GARCH models could explain most of the nonlinearities. More recently, Abhyankar et al. (1997) observed the presence of nonlinear dependence and chaos in four stock market returns where there was no evidence in the low dimensional chaotic processes. By using raw and filtered returns, Barkoulas and Travlos (1998) revealed nonlinearities in the returns of the Athens Stock Exchange through the BDS test.

Papaioannou and Karytinos (1995), on the other hand, used the BDS, $\frac{R}{S}$ and Lyapunov exponent to demonstrate nonlinear, fractal and deterministic chaos preoperties of the Athens Stock Exchange. Likewise, Andreou et al. (2000) used the same methods to analyze the chaotic and fractal structure of the Greek drachma (GRD) against the four major currencies.

There are studies in literature both supporting and rejecting the presence of chaos in financial markets while others exhibit mixed results, especially for the currency markets. For instance, Bajo-Rubio et al. (1992) detected deterministic chaos, which allows short term predictions, on daily data for the (spot and forward) Spanish Peseta–U.S. Dollar exchange rates. De Grauwe et al. (1993) stated that there was no strange attractor in the DM/USD exchange rate, while the BP/USD and JPY/USD displayed chaotic behavior. Brooks (1996) confirmed the nonlinear structure for exchange rate data, yet Serletis and Shahmoradi (2004) detected no chaos property in the returns of the CAD/USD exchange rate. Similarly, Resende and Zeidan (2008) stated that there was no evidence of chaos for different exchange rates in their study.

There is also a great deal of interest in the chaotic structures of commodities in literature. For example, Fujihara and Mougoue (1997) analyzed the linear and nonlinear behaviors on three petrol futures. According to the third order moment test results, nonlinearity in the data only arises from the variance of the process. DeCoster et al. (1992) found strong evidence for the nonlinear properties of four different commodity future prices. Chatrath et al. (2001) tested the existence of low-dimensional chaotic properties in the gold and silver future markets. Karapanagiotidis (2013) examined 25 individual continuous contract commodity futures to ascertain the existence of nonlinear and nonreversible features.

The remainder of the paper is structured as follows: section two provide literature reviews, section three give theoretical information about the BDS, the largest Lyapunov exponent and the fractal dimension, and section four present the findings of the econometrics tests.

3. ECONOMETRIC METHODOLOGY

3.1. BDS Test

In this study, we used the BDS test to test nonlinear dependence in the data, as designed by Brock et al. (1987, 1996). The null hypothesis of this test states that time series belong to a white noise, i.i.d, stochastic process. Rejection of the null hypothesis means that a time series has nonlinear dependence and chaotic features.

Although it is based on the correlation dimension suggested by Grassberger and Procaccia (1983), this model uses the correlation function instead. If u_t is a stochastic process and m is an embedding dimension, for ε >0, following the definition of Caporale et al. (2005) the correlation function is defined as follows:

$$C_{m,n}(\varepsilon)=\left[\frac{1}{\binom{\bar{n}}{2}}\right]\sum_{1\le s\le t\le n}\sum\chi_{\varepsilon}\left(\left\|u_s^m-u_t^m\right\|\right) \quad (1)$$

$$C_m(\varepsilon)=\lim_{n\to\infty}C_{m,n}(\varepsilon) \quad (2)$$

where $\bar{n}=n-(M-1),\chi_{\varepsilon}(\ldots)$ is the symmetric indicator kernel, with $\chi_{\varepsilon}(z,w)=1$, if $\|z-w\|<\varepsilon$ and 0 otherwise and if ‖. ‖ is the max-norm. The correlation function is the measure of the sequential pattern's frequency that exists in the data, and provides more efficient estimations than the correlation dimension for high dimensional chaos modeling. The BDS test statistic can be stated as follows:

$$W_{m,n}(\varepsilon)=\sqrt{n}\,\frac{\left[C_{m,n}(\varepsilon)-C_1(\varepsilon)^m\right]}{\sigma_m(\varepsilon)} \quad (3)$$

where $W_{mn}(\varepsilon)$ is BDS test statistic and $\sigma_m(\varepsilon)$ denotes the standard deviation of the $C_{mn}(\varepsilon)$ value. In the test process, two parameters, m and ε, are determined by the user. Where m is the embedding dimension and ε is the maximum differences between observations' pairs. As stated by Brock et al. (1987, 1996), there are two steps in the test process. First, the linear structure of time series is filtered using a fitted model from the linear ARMA or GARCH family. Yielding the nonlinear residuals after which the BDS test is applied to these residuals.

3.2. Largest Lyapunov Exponent

Lyapunov exponent measures the sensitivity to the initial conditions in a dynamic system. In other words, Lyapunov exponent quantifies the average rate of convergences and divergences of typical trajectories in dynamical systems (Gençay and Dechert, 1996). There are a few methods for calculating Lyapunov exponent: maximum Lyapunov exponent, largest Lyapunov exponent, local Lyapunov exponent and Lyapunov spectrum. Lyapunov spectrum is more convenient for continuous differential systems, while maximum Lyapunov is suitable for discontinuous differential systems. Local Lyapunov exponent estimates the local predictability around a X_0 point in the phase space, while the largest Lyapunov exponent provides a measure for the total predictability of a system (Dai and Han, 2012). Following the definition of Bailey (1996) the largest Lyapunov exponent statistics can be presented as follows: let x_t denotes a time series produced by a nonlinear autoregressive process, and X_t is systems trajectory ($X_t=F(X_{t-1})+E_t$), in this case the largest Lyapunov exponent is

$$\lambda=\lim_{n\to\infty}\frac{1}{n}\ln\left\|\left(J_{n-1}J_{n-2}\ldots J_0\right)u\right\| \quad (4)$$

where J_t denotes Jacobian matrice, $\|\cdots\|$ is a vector norm and $u=(0,1,\ldots,0)^T$. In a one dimensional system, Lyapunov exponent can have three different values (Brown, 1997),

$\lambda_t<0$: orbit is stable and periodic

$\lambda_t=0$: orbit is stable in a marginal way

$\lambda_t>0$: orbit is chaotic.

3.3. Fractal Dimension: Box-Counting Method

Let us consider a process to define the Box-Counting method: s denotes the side length of a box. In this case, $N(s)$ shows the needed box number in order to cover a square shape. As it is understood when s is smaller, $N(s)$ is larger. The relationship between s and $N(s)$ can be explained in the following way:

$$N(s)=k\left(\frac{1}{s}\right)^D \quad (5)$$

where k is a constant and it is not linked to box size, D demonstrates the Box-Counting dimension value and describes the scaling of self-similarity. The logarithm of both sides of the equation yields a more useful form:

$$logN(s)=logk+D\log\left(\frac{1}{s}\right) \quad (6)$$

where D presents the slope of the equation. When $logN(s)$ and $\log\left(\frac{1}{s}\right)$ are regressed using different s values, the slope of the obtained line shows the fractal dimension (Feldman, 2012). When D=2, it is Brownian motion, D=1.5 is the Brownian trace, that is the roughest surface, and finally $D=1$ demonstrates an Euclidean surface rather than a fractal. The fractality property appears when $1<D<2$ (Scholz and Aviles, 1986).

4. EMPIRICAL ANALYSIS

In this section, we analyze the nonlinearity, sensitivity to initial conditions and fractality features of stock market returns of BRIC-T. As these properties are parameters of chaotic behavior, findings will provide information about the chaotic structure of the BRIC-T countries' stock index returns. The data used is the daily log returns of stock indexes of these countries between 10/01/1997 and 01/30/2014. BRIC-T countries and their index codes are as follows: Brazil (Bovepsa), Russia (MICEX), India (NSEI) China (SSECI) and Turkey (BIST100).

4.1. BDS Test

In order to conduct the BDS test, the linear structure of the series must be filtered using a best fit model as suggested by Brock et al. (1987, 1996). Despite the fact that long memory is one of the important properties of fractality, which is another feature of the chaotic time series, aside from the studies of Saadi et al. (2006) and Grane and Veiga (2008) most literature fails to consider the long memory property in chaos analysis.

Table 1 demonstrates the test results of the Rescaled Range (Henceforth, $\frac{R}{S}$) and Smith's (2005) modified Geweke-Porter-Hudak (GPH) (1983) analysis. Both methods indicate that all of the return series have long memory features. Considering this fact we used three long memory models in the filtering process of return series: ARFIMA(p,d,q), FIGARCH (p,d,q) and FIEGARCH (p,d,q). Figure 1 shows the theoretical (blue line) and empirical (red line) Hurst exponent results.

The upward deviations of the empirical line from the theoretical line are evidence of the long memory property of the returns. The strong deviations in the index returns of Turkey and China match the empirical Hurst exponent findings seen in Table 1. As stated by Mandelbrot (1972), Hurst exponent values between 0.5 and 1 indicate the existence of long memory property. In Table 1, Smith's modified GPH test results for the fractional differencing parameter (d) are also presented. d values between 0 and 0.5 are evidence for long memory. Consequently, according to the Hurst exponent and Smith's modified GPH test, there are significant signs of long memory in return series. This information was used in the fitting procedure to obtain residuals.

The best fitting model results exhibited in Table 2 were determined using the Akaike Information Criteria and log-likelihood statistics. While the ARFIMA test results indicate no long memory in the stock index returns of Turkey and India at 95% confidence level, FIGARCH (p,d,q) and FIEGARCH (p,d,q) results show that the conditional variance has long memory for all the index returns.

After the filtration of the returns by ARFIMA (p,d,q), FIGARCH (p,d,q) and FIEGARCH (p,d,q) models, the BDS test was applied on the residuals obtained from these models. As can be seen from Table 3, three different embedding dimensions and four different epsilon values were used in the BDS test procedure. BDS tests were conducted for the log return series before the filtered ones, and all test values were statistically significant at 95% confidence level. These results rejected the null hypothesis, which states that the series comes from the i.i.d processes, yet this information is not significant enough to accept the existence of chaos as it is obtained from log return series. On the other hand, these results are not enough to reject the existence of the Efficient Market Hypothesis. According to the Efficient Market Hypothesis of Fama (1965), in an efficient market, prices fully reflect all available information; that is, residuals of the return model cannot be predicted. Technically speaking, if lnP_t is the logarithm of asset price at time t, the following equation can be applied: $lnP_{t+1}=lnP_t+e_{t+1}$ where the asset price at $t+1$ consists of the price at time t and with the new information arriving at time $t+1$. In this case, the price change can be stated as $e_{t+1}=lnP_{t+1}=lnP_t$. Here, e_{t+1} has no autocorrelation, that is e_{t+1}, should be independent from past price changes. If asset returns are random, asset price changes follow a random walk.

Table 1: Rescaled range and Smith's Modified GPH test results

Country	Hurst exponent	Smith's modified GPH test
Turkey	0.5751	0.3135** (0.0474)
Brazil	0.5217	0.4398** (0.0474)
Russia	0.5560	0.2949** (0.0474)
India	0.5446	0.2740** (0.0474)
China	0.6119	0.2299* (0.1186)

*,**indicate the significancy at 90% and 99% confidence level, respectively. Standard errors are within the parenthesis. k=4 in GPH test. GPH: Geweke-Porter-Hudak

After the test of log returns, the BDS test was applied on the residuals of the ARFIMA (p,d,q) model. Theoretically, the series are expected to be linearly independent, however all test statistics are significantly larger than the critical values for all of the confidence intervals. Therefore, the null hypothesis of the BDS test for the residuals of the ARFIMA (p,d,q) model can be rejected, and the nonlinear dependence can be attributed to the nonlinear structure of the residuals. As stated by Brock et al. (1993), if nonlinearity stems from a non-deterministic process (GARCH), we can mention the absence of chaos. Hence the BDS test can be applied on the residuals of models from the GARCH family. Rejection of the null hypothesis of this test means that conditional heteroskedasticity is not the source of nonlinearity in the returns, therefore nonlinearity may arise from other factors including chaos. Hence, in order to find out the source of nonlinearity in the ARFIMA (p,d,q) model, we also examined standardized residuals of the FIGARCH (p,d,q) and FIEGARCH (p,d,q) models through the BDS test.

As seen in Table 4, there are statistically significant findings for all index returns in the FIEGARCH models' residuals, whereas only one country exhibits statistical significance in the residuals of FIGARCH model. Another important point is that the results for each country have different significant embedding dimension levels. While significant results are generally in the high embedding dimension levels for Russia and China, Brazil and India's results are clustered in the low dimension levels. Russia and China's findings corroborate the results of Abhyankar et al. (1997) they obtained chaos features for high dimension values. In short, for the standardized residuals of FIEGARCH model, the null hypothesis of the BDS test is rejected at 95% confidence level for all countries at different dimension levels. This suggests that the nonlinear structure of the returns is not explained by the FIEGARCH model, so the nonlinearity results arise from other

Table 2: Best fitting model estimates

		ARFIMA	FIGARCH	FIEGARCH
Turkey	Model values	2.*d*.2	1.*d*.1	1.*d*.1
	Coefficient	0.0142	0.4036**	0.7060**
	Std	(0.0130)	(0.0532)	(0.0487)
Brazil	Model values	1.*d*.2	1.*d*.0	1.*d*.1
	Coefficient	−0.0468*	0.4142**	0.6722**
	Std	(0.0208)	(0.0707)	(0.0319)
Russia	Model values	3.*d*.3	0.*d*.2	1.*d*.0
	Coefficient	0.0451**	0.3610**	0.6328**
	Std	(0.0139)	(0.0453)	(0.0496)
India	Model values	1.*d*.2	1.*d*.1	1.*d*.0
	Coefficient	0.0482	0.4951**	0.4847**
	Std	(0.0517)	(0.0654)	(0.0390)
China	Model values	1.*d*.1	1.*d*.1	1.*d*.0
	Coefficient	0.0825*	0.3721**	0.5711**
	Std	(0.0364)	(0.0871)	(0.0592)

*,** indicate the significancy at 95% and %99 confidence level, respectively. Standard errors are within the parenthesis

Figure 1: Rescaled range test Hurst exponent estimates

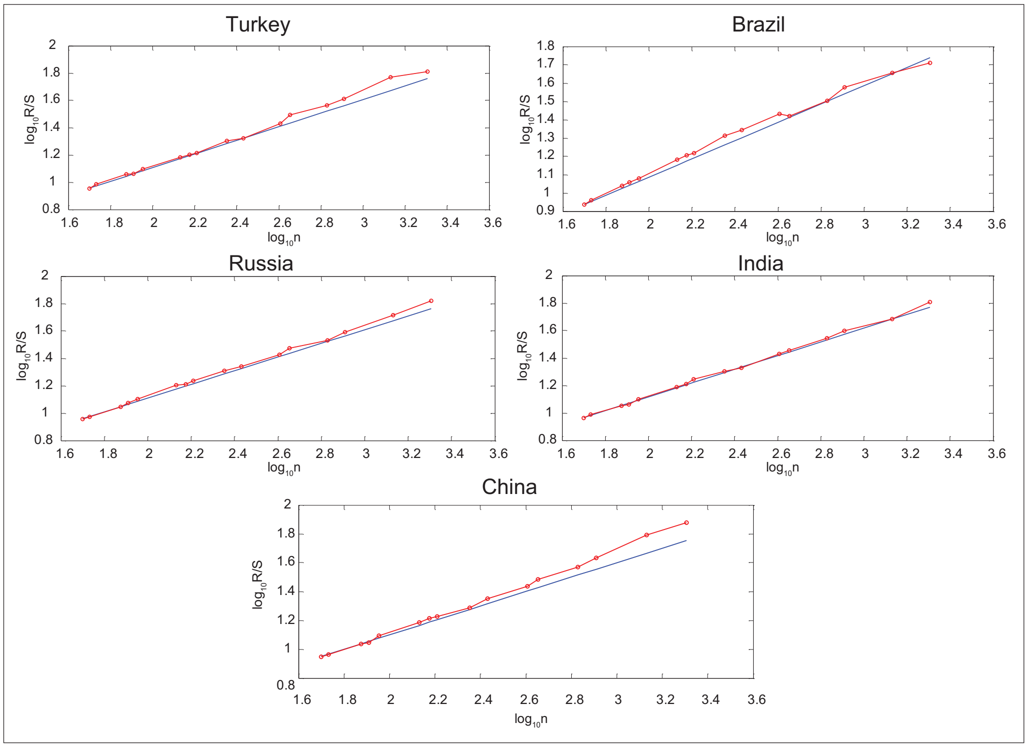

Table 3: BDS test of returns and residuals of ARFIMA model

Country	ε/σ	BDS Log returns results			BDS ARFIMA fitting results		
		m=2	*m*=5	*m*=8	*m*=2	*m*=5	*m*=8
Turkey	0.5	14.0292**	34.2963**	70.3406**	14.0292**	34.2963**	71.6819**
	1	15.2847**	31.136**	49.0258**	15.2847**	31.136**	49.9179**
	1.5	15.4016**	27.0037**	35.7933**	15.4016**	27.0037**	36.1168**
	2	15.8224**	23.6152**	27.9697**	15.8224**	23.6152**	28.0329**
Brazil	0.5	7.311**	14.9607**	23.7955**	7.311**	14.9607**	23.8807**
	1	10.8217**	18.1907**	25.5475**	10.8217**	18.1907**	25.6673**
	1.5	13.8493**	21.2581**	26.1113**	13.8493**	21.2581**	26.078**
	2	15.6381**	23.2788**	26.3842**	15.6381**	23.2788**	26.3603**
Russia	0.5	16.3837**	34.1065**	72.0958**	16.3837**	34.1065**	73.2355**
	1	19.543**	32.7956**	47.7675**	19.543**	32.7956**	48.2301**
	1.5	20.3163**	29.9057**	36.7613**	20.3163**	29.9057**	36.5113**
	2	19.2142**	26.3161**	29.889**	19.2142**	26.3161**	29.4413**
India	0.5	10.5649**	20.2006**	37.3359**	10.5649**	20.2006**	38.1615**
	1	13.1284**	22.7221**	34.0631**	13.1284**	22.7221**	34.1938**
	1.5	14.5988**	22.9793**	29.35**	14.5988**	22.9793**	29.5748**
	2	14.6799**	22.7112**	26.5894**	14.6799**	22.7112**	26.8381**
China	0.5	7.2405**	16.0039**	24.4699**	7.2405**	16.0039**	22.9552**
	1	8.8242**	17.3824**	22.1655**	8.8242**	17.3824**	21.561**
	1.5	10.2831**	18.1679**	21.9768**	10.2831**	18.1679**	21.4158**
	2	9.644**	17.2875**	20.3925**	9.644**	17.2875**	19.9873**

The 5% and 1% critical values for the BDS statistic are 1.960 and 2.575, respectively. BDS: Brock-Dechert-Scheinkman

factors including chaos. These results also support the findings of Brooks (1996) regarding emerging market studies.

4.2. Largest Lyapunov Exponent Test

Although the BDS test results provide information about the nonlinear structure of the returns, to better understand the sensitivity to initial conditions which is another parameter of chaos, we applied the largest Lyapunov exponent test on the log return series of all the indexes. In spite of the fact that there are different algorithms to conduct the Lyapunov exponent test, we preferred the Rosenstein et al. (1993) method due to the flexibility provided. This method is fast, easy to implement, and robust to different levels of embedding dimension, sample size and reconstruction delay. As no prior knowledge exists regarding the system dimension, we used three different embedding dimension levels (*m*) and reconstruction delays (*j*). Results are shown in Table 5.

Similar to the study of Rosenstein et al. (1993), the most significant results are obtained when the reconstruction delay is 1. As Table 5 shows, the standard errors for *j*=1 have the lowest values in comparison to the other delays. Unlike Rosenstein et al. (1993), who obtained the best results under lower embedding dimension levels, in our study we observed that when the embedding dimension increases, standard errors of the statistics decrease, and the most significant statistics are obtained when j=1 and m=7. As can be seen, when j=1 and m=7, all of the largest Lyapunov exponent statistics are positive. Although the coefficients are relatively close to zero due to the positive values, we can still infer to the existence of chaos.

4.3. Fractal Dimension Test: Box-Counting Method

As stated by Peters (1991), the most important components of chaotic dynamical systems are sensitivity to initial conditions and the fractal dimension. In order to adequately define chaos, we should clearly identify these elements. A chaotic dynamical system displays trajectories that converge to a strange attractor. We can obtain the effective number of degrees of freedom for this attractor via the fractal dimension and explain the complexity of the system. Although details of the trajectory are sensitive to initial conditions, the geometric structure of the strange attractor is resistant (Theiler, 1990). When the fractal dimension is larger, the chaotic structure of the system will increase. In other words, the fractal dimension is a measure of chaoticity of a system (Gündüz, 2004). We presented the fractal dimension values of all of the indexes via the Box-Counting method below.

Values in Table 6 show that the fractal dimensions (D_F) of each index are between 1 and 2, meaning the index values perform a fractal structure. Nevertheless, it is clear that all of these values are close to 1 rather than 2, suggesting that the chaoticity levels of the indexes are relatively low. These findings also support the previous results from the BDS and largest Lyapunov exponent tests. Figure 2 presents an alternative view to the results of the BDS, largest Lyapunov exponent and Box-Counting tests. Accordingly, the largest Lyapunov and Box-Counting tests statistics match up with each other in terms of high statistical values. Despite that the BDS statistics demonstrate a different ordering; the largest value belongs to Brazil while the smallest belongs to China.

5. CONCLUSION AND POLICY IMPLICATIONS

Determining the existence of chaos in financial markets requires the re-examination of conventional finance theory. In fact, there is ample literature which presents the gaps of conventional theory and its dependence on i.i.d processes. By means of these studies several stylized facts were revealed by different researchers, such as dependence in financial asset returns, fat tails in return distributions and volatility clustering. Along these lines, the present study investigated the chaotic features of five leading emerging stock markets: BRIC-T in order to contribute the current literature. Chaotic structure of the BRIC-T countries' stock markets were examined in the context of nonlinearity, sensitivity to initial conditions and fractality using the BDS, largest Lyapunov exponent and Box-Counting methods. As financial time series may contain long memory, the ARFIMA, FIGARCH and FIEGARCH methods were used in the filtering process to incorporate long memory property into conditional mean and conditional variance. After filtering the linear structure of log return series the BDS test was conducted on residuals to identify nonlinear dependence of the series. ARFIMA and FIEGARCH models demonstrated that nonlinear structure of the returns cannot be explained through these models. When long memory and the asymmetric structure of volatility are taken into account, the BRIC-T stock index returns have nonlinear dependence which attributed to other factors including chaos. Largest Lyapunov exponent test, which was conducted to assess sensitivity to initial conditions, showed that the most significant results occurred where under j=1 and m=7. Although all test statistics were close to zero, as results were positive, they were seen as an indicator of chaos.

Table 4: BDS test of residuals of FIGARCH and FIEGARCH model

Country	ε/σ	BDS FIGARCH fitting results			BDS FIEGARCH fitting results		
		m=2	m=5	m=8	m=2	m=5	m=8
Turkey	0.5	−1.3485	−0.4293	−0.8017	−3.0544	−0.4695	5.7674**
	1	−1.2027	−0.8883	−0.9701	−2.674**	−1.0844	−0.2649
	1.5	−0.3347	−0.325	−0.6122	−1.8670	−0.7329	−0.5023
	2	0.9333	0.8801	0.4529	−0.5915	0.3702	0.2289
Brazil	0.5	−1.0000	−1.2935	−1.7215	−5.323**	−1.6620	13.579**
	1	−1.0838	−1.8173	−1.2902	−5.866**	−3.142**	−0.7403
	1.5	−0.9297	−1.7910	−1.3051	−5.572**	−3.020**	−1.0357
	2	−0.4407	−1.3949	−1.1064	−4.445**	−2.5257*	−1.0543
Russia	0.5	0.0810	−0.3842	1.3395	0.5540	1.7553	3.8709**
	1	−0.2766	−0.5688	0.3176	0.5209	2.2441*	3.6511**
	1.5	−0.3637	−0.4856	0.2484	0.7070	2.7021*	3.9562**
	2	−0.5028	−0.1672	0.4280	0.6777	2.938**	3.9958**
India	0.5	−1.5100	−1.4019	−0.1544	−3.087**	−2.997**	−1.6142
	1	−1.2274	−1.1099	−0.1894	−2.812**	−2.744**	−1.2891
	1.5	−1.1403	−0.8805	−0.1349	−2.5235*	−2.4774*	−0.8850
	2	−0.4973	−0.1992	0.2417	−1.6815	−1.4829	−0.0068
China	0.5	−0.4572	3.1745**	4.384**	−1.2330	1.9758*	3.1096**
	1	−0.9763	1.1380	0.9824	−0.8251	1.2063	1.5023
	1.5	−0.4084	0.4274	0.0703	−0.0308	1.0853	1.3484
	2	−0.0779	0.2372	−0.2809	0.5979	1.3243	1.4811

The 5% and 1% critical values for the BDS statistic are 1.960 and 2.575, respectively. BDS: Brock-Dechert-Scheinkman

Figure 2: Summary Statistics of BDS (ε/σ and *m*=8), Largest Lyapunov Exponent (*j*=1 *m*=7) and Box-Counting Tests

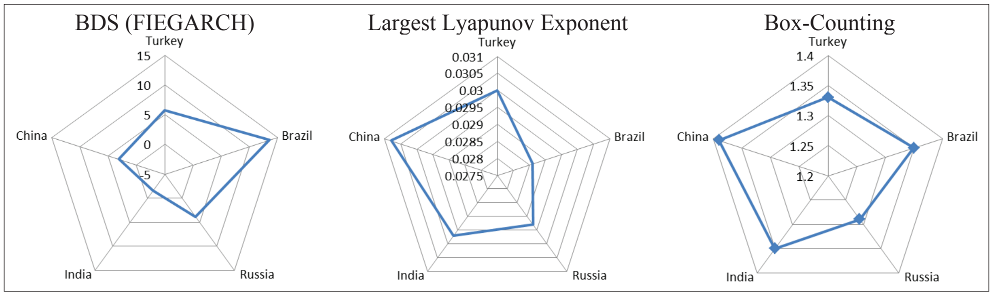

Table 5: Largest Lyapunov exponent test results

		m=3	*m*=5	*m*=7
Turkey	*J*=1	0.0334* (0.0129)	0.0301** (0.0083)	0.0300** (0.0062)
	J=5	0.0278* (0.0132)	0.0197* (0.0093)	0.0135 (0.0080)
	J=10	0.0230 (0.0138)	0.0131 (0.0098)	0.0092 (0.0082)
Brazil	*J*=1	0.0333* (0.0131)	0.0298** (0.0084)	0.0286** (0.0063)
	J=5	0.0286* (0.0135)	0.0192 (0.0094)	0.0126 (0.0081)
	J=10	0.0229 (0.0138)	0.0129 (0.0099)	0.0090 (0.0082)
Russia	*J*=1	0.0330* (0.0131)	0.0297** (0.0082)	0.0293** (0.0063)
	J=5	0.0288* (0.0134)	0.0198* (0.0093)	0.0131 (0.0080)
	J=10	0.0231 (0.0138)	0.0130 (0.0098)	0.0086 (0.0082)
India	*J*=1	0.0334* (0.0130)	0.0301** (0.0082)	0.0297** (0.0061)
	J=5	0.0288* (0.0133)	0.0199* (0.0094)	0.0133 (0.0080)
	J=10	0.0234 (0.0139)	0.0137 (0.0098)	0.0093 (0.0082)
China	*J*=1	0.0337* (0.0137)	0.0311** (0.0090)	0.0308** (0.0066)
	J=5	0.0291* (0.0137)	0.0198 (0.0096)	0.0134 (0.0084)
	J=10	0.0237 (0.0141)	0.0138 (0.0101)	0.0097 (0.0084)

*,** indicate the significancy at % 95 and %99 confidence level, respectively. Standard errors are within the parenthesis

Table 6: Box-counting method results

	Turkey	Brazil	Russia	India	China
D_F	1.33	1.35	1.29	1.35	1.39

Consistent with the previous results, findings of the fractal dimension test also supported the existence of chaos at a low degree. Consequently, we have concluded that BRIC-T stock markets have chaotic features at low to moderate intensity levels. Determining the presence of chaos, even at low levels, demand considering nonlinear relations and fractality in financial markets and financial modeling.

REFERENCES

Abhyankar, A., Copeland, L.S., Wong, W. (1997), Uncovering nonlinear structure in real time stock market indexes: the S&P 500, the DAX, the Nikkei 225, and the FTSE-100. Journal of Business and Economics Statistics, 15, 1-14.

Andreou, A.S., Karytinos, A., Pavlides, G. (2000), Non-linear time-series analysis of the Greek exchange-rate market. International Journal of Bifurcation and Chaos, 10(7), 1729-1758.

Bailey, B.A. (1996), Local Lyapunov exponents: predictability depends on where you are. In: Barnett, W.A., Kirman, A.P., Salmon, M., editors. Nonlinear Dynamics and Economics. Cambridge: Cambridge University Press, p345-359.

Bajo-Rubio, O., Fernández-Rodríguez, F., Sosvilla-Rivero, S. (1992), Chaotic behaviour in exchange-rate series: first results for the Peseta-U.S. Dollar case'. Economics Letters, 39, 207-211.

Barkoulas, J., Travlos, N. (1998), Chaos in an emerging capital market? The aase of the Athens stock exchange. Applied Financial Economics, 8, 231-243.

Barnett, W.A., Chen, P. (1988), The aggregation-theoretic monetary aggregates are chaotic and have strange attractors: an econometric application of mathematical Chaos. In: Barnett, W.A., Berndt, E., White, H., editors. Dynamic Econometric Modeling. Cambridge: Cambridge University Press. p199-245.

Brock, W.A. (1986), Distinguishing random and deterministic systems: abridged version. Journal of Economic Theory, 40(1), 168-195.

Brock, W.A., Dechert, W.D., Scheinkman, J.A., LeBaron, B. (1996), A test for independence based on the correlation dimension. Econometric Reviews, 15, 197-235.

Brock, W.A., Dechert, W.D.,Scheinkman, J. (1987), A Test for Independence Based on the Correlation Dimension, Madison: Department of Economics, University of Wisconsin, SSRI Working Paper, 8702.

Brock, W.A., Hsieh, D.A., LeBaron, B. (1993), Non-Linear Dynamics, Chaos, and Instability: statistical Theory and Economic Evidence. Cambridge: MIT Press.

Brockett, P.L, Hinich, M.J., Patterson, D.M. (1988), Bispectral based tests for detection of Gaussianity and linearity in time series. Journal of the American Statistical Association, 83(403), 657-664.

Brooks, C. (1996), Testing for non-linearity in daily sterling exchange rates. Applied Financial Economics, 6, 307-317.

Brown, T.A. (1997), Measuring Chaos using the Lyapunov exponent. In: Kiel, L.D., Elliott, E.W., editors. Chaos Theory in the Social Sciences: Foundations and Applications. USA: University of Michigan Press.

Caporale, G.M., Ntantamis, C., Pantelidis, T., Pittis, N. (2005), The BDS test as a test for the adequacy of a GARCH(1,1) specification: a Monte Carlo study. Journal of Financial Econometrics, 3(2), 282-309.

Chatrath, A., Bahram, A., Todd, S. (2001), Nonlinear dependence in gold and silver futures: is it Chaos? Business Administration Faculty

Publications and Presentations, 45(2), 25-32.
Dai, L., Han, L. (2012), Characteristics diagnosis of nonlinear dynamical systems. In: Luo, A.C., Machado, J.A., Baleanu, D., editors. Dynamical Systems and Methods. London: Springer. p85-104.
DeCoster, G.P., Labys, W.C., Mitchell, D.W. (1992), Evidence of Chaos in commodity futures prices. The Journal of Futures Markets, 12(3), 291-305.
DeCoster, G.P., Mitchell, D.W. (1991), Nonlinear monetary dynamics. Journal of Business and Economic Statistics, 9, 455-462.
De Grauwe, P., Dewachter, H., Embrechts, M. (1993), Exchange rate theory: chaotic models of foreign exchange markets. Oxford: Blackwell, United Kingdom.
Fama, E.F. (1965), The behavior of stock-market prices. The Journal of Business, 38(1), 34-105.
Feldman, D.P. (2012), Chaos and Fractals: An Elementary Introduction. United Kingdom: Oxford University Press.
Fujihara, R.A., Mougoue, M. (1997), An examination of linear and non-linear causal relationships between price variability and volume in petroleum futures markets. Journal of Futures Market, 4, 385-416.
Williams, G.P. (1997), Chaos Theory Tamed. Washington: Joseph Henry Press.
Gençay, R., Dechert, W.D. (1996), He identification of spurious. Lyapunov exponents in Jacobian algorithms. Studies in Nonlinear Dynamics and Econometrics, 1(3): 145-154.
Geweke, J., Porter-Hudak, S. (1983), The estimation and appication of long memory time series models. Journal of Time Series Analysis, 4(4), 221-238.
Grane, A., Veiga, H. (2008), Accurate minimum capital risk requirements: a comparison of several approaches, Statistic and Econometric Series 13, Working Paper, 07-47. p1-22.
Grassberger, P., Procaccia, I. (1983), Measuring the strangeness of strange attractors. Physica D, 9, 189-208.
Gündüz, G. (2004), Anaksimenes'te ve Günümüzde Kaos Anlayışı, Mantık, Matematik ve Felsefe II. Ulusal Sempozyumu, Bildiriler Kitabı, Turkey.
Ban, H.H., Shachrnurove, Y. (2002), Chaos Theory and its Application, Caress Working Paper, Center for Analytic Research in Economics and the Social Sciences, 2002.
Hinich, M.J., Patterson, D.M. (1985), Evidence of nonlinearity in daily stock returns. Journal of Business and Economic Statistics, 3, 69-77.
Hsieh, D.A. (1991), Chaos and nonlinear dynamics: application to financial markets. The Journal of Finance, 46(5), 1839-1877.
Hsieh, D. (1989), Testing for nonlinearity in daily foreign exchange rate changes. Journal of Business, 62, 339-368.
Karapanagiotidis, P. (2013), Empirical evidence for nonlinearity and irreversibility of commodity futures prices. DWAE, 1st Annual Doctoral Workshop on Applied Econometrics. p1-116.
Cuthbertson, K. (1999), Quantitative Financial Economics: Stocks, Bonds and Foreign Exchage. United Kingdom: John Wiley & Sons.
Lui, T., Granger, C.W.J., Heller, W.P. (2001), Using the correlation exponent to decide whether an economic series is chaotic, Essays in econometrics. New York: Cambridge University Press. p188-207.
Mandelbrot, B.B. (1972), Statistical methodology for nonperiodic cycles from covariance to R/S analysis. Annals of Economic and Social Measurement, 1, 259-290.
Papaioannou, G., Karytinos, A. (1995), Nonlinear time series analysis of the stock exchange: the case of an emerging market. International Journal of Bifurcations and Chaos, 5, 1557-1584.
Peters, E.E. (1991), Chaos and Order in the Capital Markets: A New View of Cycles, Prices, and Market Volatility'. New York: John Wiley & Sons.
Resende, M., Zeidan, R. (2008), Expectations and Chaotic Dynamics: Empirical Evidence on Exchange Rates. Economics Letters, 99, 33-35.
Rosenstein, M.T., Collins, J.J., Luca, C.J.D. (1993), A practical method for calculating largest Lyapunov exponents from small data sets. Phys. D, 65(7), 117-134.
Saadi S., Gandhi D., Dutta S. (2006), Testing for nonlinearıty & modeling volatility in emerging capital markets: the case of Tunisia. International Journal of Theoretical and Applied Finance, 09(07), 1021-1050.
Scheinkman, J.A., LeBaron, B. (1989), Nonlinear dynamics and stock returns. Journal of Business, 62, 311-337.
Scholz, C.H., Aviles, C.A. (1986), The fractal geometry of faults and faulting, in earthquake source Mechanics. Geophysical Monograph Series, 37, 147-155.
Serletis, A., Shahmoradi, A. (2004), Absence of Chaos and 1/f spectra, but evidence of TAR nonlinearities, in the Canadian exchange rate. Macroeconomic Dynamics, 8, 543-551.
Smith, A. (2005), Level shifts and the illusion of long memory in economic time series. Journal of Business and Economic Statistics, 23(3), 321-335.
Takens, F. (1981), Detecting strange attractors in turbulence. Lecture Notes in Mathematics, 898, 366-381.
Theiler, J., (1990), Estimating the Fractal Dimension oj Chaotic Time Series, The Uncoln Laboratory Journal, 3(1), 63-86.